Bonnie and Justin created these logos for the Student Council. Their advisor tells them that the triangles are similar. How can she tell? What do you know about similar figures? What strategies can you develop to determine if triangles are similar?

**Explore How to Identify Similar Triangles**

1. Trace each logo on separate pieces of tracing paper.

2. **a)** Measure the angles in each logo. What do you notice about the corresponding angles?  
   **b)** Measure the side lengths in each logo. What do you notice about the ratios of the corresponding sides of the triangles?

**Reflect and Check**

3. **a)** What conclusions can you make about the corresponding angles of the two triangles?  
   **b)** What conclusions can you make about the corresponding sides of the two triangles?

4. **a)** What conditions do you think are necessary in order for two triangles to be similar?  
   **b)** Test the conditions on a different set of two triangles. Are the triangles similar? Discuss with a classmate why you think the triangles are, or are not, similar.
**Example 1: Identify Similar Triangles**

Determine if \( \triangle ABC \) is similar to \( \triangle EFG \).

![Diagram of \( \triangle ABC \) and \( \triangle EFG \)]

**Solution**

Similar triangles have corresponding angles that are equal in measure and corresponding sides that are proportional in length.

Compare corresponding angles:
- \( \angle A = 90^\circ \) and \( \angle E = 90^\circ \)
- \( \angle B = 37^\circ \) and \( \angle F = 37^\circ \)
- \( \angle C = 53^\circ \) and \( \angle G = 53^\circ \)

The corresponding angles are equal.

Compare corresponding sides:
\[
\frac{AB}{EF} = \frac{12}{4} = 3 \quad \frac{BC}{FG} = \frac{15}{5} = 3 \quad \frac{AC}{EG} = \frac{9}{3} = 3
\]

The corresponding sides are proportional with a scale factor of 3.

\( \triangle ABC \sim \triangle EFG \)

**Literacy Link**

Angles can be named in two ways:
- Use three capital letters. The middle letter is the vertex of the angle.
- Use only the middle letter identifying the vertex. Use a single letter when there is only one angle at a vertex.

For example, the angle at vertex \( L \) can be named \( \angle KLM \) or \( \angle L \).
Determine if each pair of triangles is similar. Show how you know.

### a)

<table>
<thead>
<tr>
<th>Q</th>
<th>2.8</th>
<th>P</th>
<th>4.2</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>3</td>
<td>4.5</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

$\angle R = 69^\circ$  
$\angle T = 69^\circ$

### b)

<table>
<thead>
<tr>
<th>A</th>
<th>2.7</th>
<th>E</th>
<th>2.5</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>3.3</td>
<td>C</td>
<td>3</td>
<td>F</td>
</tr>
<tr>
<td>70°</td>
<td>70°</td>
<td>4.1</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

$\angle A = 70^\circ$  
$\angle D = 70^\circ$

### Example 2: Use Similar Triangles to Determine a Missing Side Length

Kyle is drawing triangles for a math puzzle. Use your knowledge of similar triangles to determine

a) if the triangles are similar  
b) the missing side length

#### Solution

a) Check that $\triangle KLM$ is similar to $\triangle TUV$.

The sum of the angles in a triangle is 180°.

$\angle K = 180^\circ - 50^\circ - 85^\circ$  
$= 45^\circ$

$\angle U = 180^\circ - 85^\circ - 45^\circ$  
$= 50^\circ$

Compare corresponding angles:

$\angle K = 45^\circ$ and $\angle T = 45^\circ$  
$\angle L = 50^\circ$ and $\angle U = 50^\circ$  
$\angle M = 85^\circ$ and $\angle V = 85^\circ$

All pairs of corresponding angles are equal.

Therefore, $\triangle KLM \sim \triangle TUV$.

b) You can compare corresponding sides to determine the scale factor.

$$
\frac{LM}{UV} = \frac{24}{8} \quad \frac{KM}{TV} = \frac{21}{7} \quad \frac{KL}{TU} = \frac{x}{10.5}
$$

$$
= 3 \quad = 3 \quad = \frac{3}{\frac{10.5}{x}}
$$

The scale factor is 3. You can solve for the unknown length.
**Method 1: Use a Scale Factor**
Since the triangles are similar, you can use the scale factor to determine the missing length.

\[
\frac{x}{10.5} = 3
\]

\[
x = 31.5
\]

The missing side length is 31.5 units.

**Method 2: Use a Proportion**
Since the triangles are similar, you can use equal ratios to set up a proportion.

\[
\frac{KM}{TV} = \frac{KL}{TU}
\]

\[
\times 1.5
\]

\[
\frac{21}{7} = \frac{x}{10.5}
\]

\[
\times 1.5
\]

\[
x = 31.5
\]

The missing side length is 31.5 units.

---

**Show You Know**

Solve using a method of your choice.

**a)** \(\triangle GHI\) is similar to \(\triangle KLM\). What is the missing side length? Express your answer to the nearest tenth.

**b)** \(\triangle ABC\) is similar to \(\triangle EFC\). Determine the missing side length. Express your answer to the nearest tenth.
Key Ideas

• Triangles are similar if one of the following conditions holds true:
  ▪ corresponding angles are equal in measure
  ▪ corresponding sides are proportional in length

\[ \triangle DEF \text{ is similar to } \triangle ABC. \]
\[ \triangle DEF \text{ is not similar to } \triangle PQR. \]
\[ \angle D = \angle A, \angle E = \angle B, \angle F = \angle C \]
\[ \frac{DE}{AB} = \frac{3}{1.5}, \quad \frac{EF}{BC} = \frac{2.2}{1.1}, \quad \frac{DF}{AC} = \frac{2.6}{1.3} \]

• You can solve problems related to similar triangles using different methods.
  ▪ Use a scale factor.
  ▪ Use a proportion.

Check Your Understanding

Communicate the Ideas

1. If two triangles are similar, what can you say about the angles of the triangles? the side lengths of the triangles?

2. Amanda is unclear about similar triangles. She drew these two triangles and states they are similar. Is she correct? Explain.

3. Are two triangles that have equal angles and equal sides similar? Use an example to support your answer.

Practise

For help with #4 to #8, refer to Example 1 on page 147.

4. List the corresponding angles and the corresponding sides for \( \triangle PQR \) and \( \triangle TUV \).

5. What are the corresponding angles and the corresponding sides in this pair of triangles?
6. Are the triangles similar? Show how you know.

[Diagram of triangle with sides labeled 2.2, 12.5, and 6]

7. Determine if the triangles are similar. Show how you know.

[Diagram of triangle with sides labeled 7.5, 18, and 12.5]

8. Determine which pairs of triangles are similar. Use a sketch to help explain how you know.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Angles</th>
<th>Sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>△ABC</td>
<td>∠A = 90°</td>
<td>AB = 6</td>
</tr>
<tr>
<td></td>
<td>∠B = 45°</td>
<td>BC = 8.4</td>
</tr>
<tr>
<td></td>
<td>∠C = 45°</td>
<td>AC = 6</td>
</tr>
<tr>
<td>△EFG</td>
<td>∠E = 90°</td>
<td>EF = 3</td>
</tr>
<tr>
<td></td>
<td>∠F = 45°</td>
<td>FG = 4.2</td>
</tr>
<tr>
<td></td>
<td>∠G = 45°</td>
<td>EG = 3</td>
</tr>
<tr>
<td>△HIJ</td>
<td>∠H = 90°</td>
<td>HI = 9.2</td>
</tr>
<tr>
<td></td>
<td>∠I = 60°</td>
<td>IJ = 18.4</td>
</tr>
<tr>
<td></td>
<td>∠J = 30°</td>
<td>HJ = 15.9</td>
</tr>
<tr>
<td>△KLM</td>
<td>∠K = 90°</td>
<td>KL = 9</td>
</tr>
<tr>
<td></td>
<td>∠L = 45°</td>
<td>LM = 12.6</td>
</tr>
<tr>
<td></td>
<td>∠M = 45°</td>
<td>KM = 9</td>
</tr>
</tbody>
</table>

9. △STR is similar to △UWV. Determine the missing side length.

[Diagram of similar triangles with sides labeled 8, 10.5, and 73.5]

10. △CDE is similar to △GFE. What is the missing side length?

[Diagram of similar triangles with sides labeled 22.5, 18, and 7.5]

11. Draw a triangle that is similar to the one shown. Label the measurements for angles and sides on your similar triangle.

12. Sam built a ramp to a loading dock. The ramp has a vertical support 2 m from the base of the loading dock and 3 m from the base of the ramp. If the vertical support is 1.2 m in height, what is the height of the loading dock?

[Diagram of ramp with vertical support and measurements]
13. Two extension ladders are leaning at the same angle against a vertical wall. The 3-m ladder reaches 2.4 m up the wall. How much farther up the wall does the 8-m ladder reach?

![Diagram of ladders against a wall]

14. Erin, who is 1.60 m tall, casts a shadow that is 1.25 m long. Her shadow extends to the end of a tree’s shadow when she stands 4.75 m from the tree. What is the height of the tree?

![Diagram of Erin and the tree]

15. Sara was helping her father assemble a slide for the local park. He decides to reinforce the slide with an extra support beam. How long should the extra support beam be?

![Diagram of slide and support beam]

16. Peter, who is 168 cm tall, casts a 45-cm shadow. Michael, who is standing beside him, casts a 40-cm shadow. Can you tell who is taller? Use a diagram to help explain why or why not.

![Diagram of Peter and Michael with their shadows]

17. Develop a word problem that can be solved using similar triangles. Include a diagram.

18. A tourist wants to estimate the height of an office tower. He places a mirror on the ground and moves away to sight the top of the tower in the mirror.

![Diagram of the tower and the mirror]

**Extend**

a) How tall is the tower?

b) In this situation, why is the mirror reflection a better way to indirectly measure the tower than by using shadows?

19. Is it possible for the two triangles described below to be similar? Explain your reasoning.

   a) Two angles of one triangle measure 60° and 70°. Two angles of the other triangle measure 50° and 80°.

   b) Two angles of one triangle measure 45° and 75°. Two angles of the other triangle measure 45° and 60°.

20. The sides of a triangle measure 3 cm, 5 cm, and 6 cm. If the side of a similar triangle corresponding to 3 cm measures 8 cm,

   a) determine the lengths of the other sides

   b) determine the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle

21. Using a measuring tape, your shadow, and yourself, how can you determine the height of your school without actually measuring it?
22. \(\triangle WXY\) is similar to \(\triangle ZWY\). Calculate 
\(ZY\) to the nearest tenth.

```
\[
\begin{array}{c}
W \\
\hline
X \\
\hline
Z \\
\hline
Y
\end{array}
\]
```

```
\[
\begin{array}{c}
10 \text{ cm} \\
\hline
7 \text{ cm}
\end{array}
\]
```

23. Use two different sets of measurements to 
determine the area of \(\triangle KLM\).

```
\[
\begin{array}{c}
K \\
\hline
L \\
\hline
M
\end{array}
\]
```

```
\[
\begin{array}{c}
20 \text{ cm} \\
\hline
h \\
\hline
25 \text{ cm}
\end{array}
\]
```

Math Link

For your design project report, include a signature logo that features your name.

a) On a sheet of 8.5 \(\times\) 11 paper, design your logo. Include a triangle that is 
similar to the one shown. Measure all the angles and side lengths.

b) Draw a scale diagram of the logo to fit on your design project. Identify the 
scale factor you used.

Tech Link

Similarity and Scale Factors

In this activity, you can use dynamic geometry 
software to explore similarity and scale factors. 
To use this activity, go to www.mathlinks9.ca 
and follow the links.

Explore

1. Slide point X along line segment AB and 
describe what happens to the image drawing.

2. How do the measures of the corresponding 
sides of the drawing change relative to each 
other? Explain.

3. Compare the scale factor to the lengths of the 
sides of the original drawing and the image 
drawing. Create and complete a table similar to 
the one below with measurements taken at different 
locations. Discuss your findings with a classmate. 
Hint: In the table, \(m\) means the measure of.

<table>
<thead>
<tr>
<th>(mFE)</th>
<th>(mF'E')</th>
<th>(\frac{mAX}{mXB})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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